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EFFECT OF QUASI-TOTAL INTERNAL REFLECTION OF SHOCK WAVES AT AN INTERFACE BETWEEN TWO ELASTIC MEDIA

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The interaction of a spherical shock stress wave generated by a point explosion with a plane interface between two elastic media with different mechanical parameters is considered. Using the method of zero approximation of ray theory, it is shown that, after passage of a shock from a medium with smaller acoustic stiffness into a medium with greater acoustic stiffness, the stresses at the front of the refracted shock tend to infinity near the points of quasi-total internal reflection, and this can result in local failure of the media. An experiment with a two-component medium water–Plexiglas confirms the theoretically predicted phenomenon with sufficient accuracy.

Introduction. In geometrical optics there is the well-known effect of total internal reflection, which is related to the distinctive feature of refraction of light rays passing through an interface G between two isotropic transparent media with different indices of refraction n_1 and n_2. As follows from Snell's law, for a light ray that is incident at angle φ on the interface and is refracted at angle χ, the relation $n_1 \sin\varphi = n_2 \sin\chi$ holds. Therefore, $\chi = \arcsin[(n_1/n_2)\sin\varphi]$. In the case $n_1 > n_2$ for a certain value of φ, the expression in square brackets becomes equal to unity and exceeds it with further increase in φ. Since the arcsine function is not defined for arguments greater than unity, there is no refraction of light for these values of φ. In this case, the entire energy brought to the interface G by the incident wave is carried away by the reflected wave, so that light does not penetrate into the second medium (the effect of total internal reflection).

A similar effect also takes place for shock waves at interfaces between elastic media, although all phenomena are more complicated in this case.

Let P be longitudinally polarized shock waves and S be transverse waves. The subscripts 1 and 2 correspond to the waves propagating in media 1 and 2, respectively, and the minus and plus signs refer to parameters of the waves before and after their impact interaction with the plane G separating media 1 and 2.

We assume that a plane longitudinal shock wave P_{1-} (Fig. 1), propagating in medium 1, is incident at angle θ_{1-} on the plane interface G between media 1 and 2. Interaction of the wave with this plane gives rise to reflected P_{1+} and refracted P_{2+} longitudinal shock waves, which propagate in media 1 and 2, respectively. The values of the refraction angle θ_{2+} can be obtained using Snell's law for elastic media [1] $\sin\theta_{1-}/\alpha_1 = \sin\theta_{2+}/\alpha_2$, whence $\sin\theta_{2+} = \alpha_2 \sin\theta_{1-}/\alpha_1$, where α_1 and α_2 are the propagation velocities of P-waves in the corresponding media.

If $\alpha_2 > \alpha_1$, then $\sin\theta_{2+} = 1$ for some $\theta_{1-} = \arcsin(\alpha_1/\alpha_2)$, and it must be greater than unity with further increase in θ_{1-}. However, since the function $\sin\theta_{2+}$ cannot be greater than unity, the value $\theta_{1-} = \arcsin(\alpha_1/\alpha_2)$ is a limiting value, for which $\sin\theta_{2+} = 1$, $\theta_{2+} = \pi/2$, and the interaction of the

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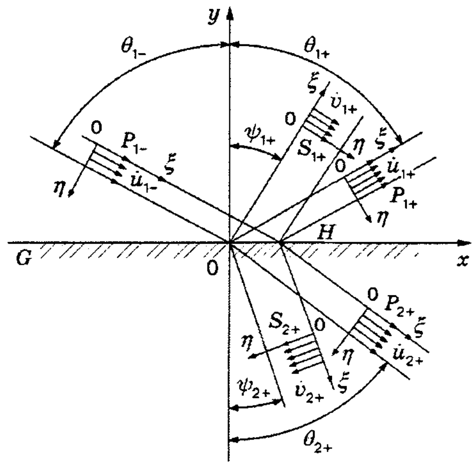

Fig. 1

wave (P_{1-}) with the boundary G follows the pattern described above. For $\theta_{1-} > \arcsin(\alpha_1/\alpha_2)$ there is no refraction of the transmitted wave (P_{2+}) because the angle θ_{2+} must be larger than $\pi/2$, but this is impossible. The point at which $\theta_{1-} = \arcsin(\alpha_1/\alpha_2)$, $\sin\theta_{2+} = (\alpha_2/\alpha_1)\sin(\arcsin(\alpha_1/\alpha_2)) = (\alpha_2/\alpha_1)(\alpha_1/\alpha_2) = 1$, and $\theta_{2+} = \pi/2$ is not a point of total internal reflection because there still exists a transverse wave (S_{2+}), whose velocity β_2 is less than the velocity α_2 and, hence, the energy of the incident wave P_{1-} is not entirely reflected from the surface G. Therefore, we call this effect quasi-total internal reflection (QTIR).

We should consider one more distinctive feature of the interaction of the incident, reflected, and refracted waves at the point H of conjugation of their root lines on the surface G with approach to the point of QTIR. Since, for $\alpha_2 > \alpha_1$, $\sin\theta_{1-} = \alpha_1/\alpha_2$ at this point, we have $\alpha_1/\sin\theta_{1-} = \alpha_2$. However, geometric constructions show that the left side $\alpha_1/\sin\theta_{1-}$ of the last equality represents the velocity v of motion of the point H along the surface G. At the point of QTIR, the velocity v, which essentially is the velocity of propagation of the perturbation of the second medium along the boundary G, coincides with the velocity α_2 of propagation of the wave P_2 in this medium. It is known that such situations in wave mechanics are critical since they are accompanied by infinite increase in the amplitude of the generated wave. Therefore, we can expect that unbounded growth of strains (and stresses) at the point of QTIR occurs in our case too. To verify this assumption, we carried out theoretical and experimental studies.

1. Theoretical Study of the QTIR Effect. To solve the problem of propagation of unsteady waves in elastic media, we choose the ray method [1–3], which allow us to obtain a sufficiently accurate solution at the wave fronts using a ray series.

In an analysis of shock waves, the most important information on the dynamic effect of a wave on the objects considered is contained in the zeroth term, which determines the jump of the sought function at the wave front and the main part of the momentum transferred by the wave. Thus, only the zeroth term in this expansion should be taken into account. In this case, the problem can be substantially simplified by considering only short impulsive waves, for which it is not necessary to construct the entire longitudinal shock-wave profile. To solve this problem, it is convenient to use methods developed in the stereomechanical theory of impact [4].

Under the assumptions formulated above, the directions of the rays of waves reflected and refracted at the interface G are determined using Snell's law [3]. In the case of a longitudinally polarized incident wave, it is written as

$$\frac{\sin\theta_{1-}}{\alpha_1} = \frac{\sin\theta_{1+}}{\alpha_1} = \frac{\sin\psi_{1+}}{\beta_1} = \frac{\sin\theta_{2+}}{\alpha_2} = \frac{\sin\psi_{2+}}{\beta_2}, \tag{1.1}$$

where θ and ψ are the angles of incidence of the longitudinal and transverse waves, respectively.

The intensities of the waves transformed at the interface are determined from the condition of conservation of momentum in the ray tubes of the corresponding shock waves before and after their interaction (Fig. 1) and from the condition of continuity of the velocity vector $\dot{\boldsymbol{U}}$ of elastic particles with longitudinal $\dot{\boldsymbol{u}}$ and transverse $\dot{\boldsymbol{v}}$ components at the plane interface G. These conditions have the form of linear algebraic equations [3]:

$$\begin{gathered}\alpha_1\rho_1\dot{u}_{1+}\sin\theta_1\cos\theta_1+\beta_1\rho_1\dot{v}_{1+}\cos^2\psi_1+\alpha_2\rho_2\dot{u}_{2+}\sin\theta_2\cos\theta_2\\ -\beta_2\rho_2\dot{v}_{2+}\cos^2\psi_2=\alpha_1\rho_1\dot{u}_{1-}\sin\theta_1\cos\theta_1,\\ \alpha_1\rho_1\dot{u}_{1+}\cos^2\theta_1-\beta_1\rho_1\dot{v}_{1+}\sin\psi_1\cos\psi_1-\alpha_2\rho_2\dot{u}_{2+}\cos^2\theta_2\\ -\beta_2\rho_2\dot{v}_{2+}\sin\psi_2\cos\psi_2=-\alpha_1\rho_1\dot{u}_{1-}\cos^2\theta_1,\\ \dot{u}_{1+}\sin\theta_1+\dot{v}_{1+}\cos\psi_1-\dot{u}_{2+}\sin\theta_2+\dot{v}_{2+}\cos\psi_2=-\dot{u}_{1-}\sin\theta_1,\\ \dot{u}_{1+}\cos\theta_1-\dot{v}_{1+}\sin\psi_1+\dot{u}_{2+}\cos\theta_2+\dot{v}_{2+}\sin\psi_2=\dot{u}_{1-}\cos\theta_1.\end{gathered} \tag{1.2}$$

Here ρ_i are the densities of the media, and α_i and β_i are the propagation velocities of the longitudinal and transverse waves, respectively. This relations make it possible to express the velocities $\dot{u}_{1+}$, $\dot{v}_{1+}$, $\dot{u}_{2+}$, and $\dot{v}_{2+}$ of the elements of the elastic media via the velocity $\dot{u}_{1-}$ in the incident wave and to construct the reflected and refracted waves.

Using the expressions for the stress tensor in radial coordinates (ξ, η, ζ), it is not difficult to find the relation between the quantities $\dot{u}_{1+}$, $\dot{v}_{1+}$, $\dot{u}_{2+}$, and $\dot{v}_{2+}$ and the jumps of stresses at the shock front $\sigma_{\xi\xi} = -\rho\alpha\dot{u}$ and $\sigma_{\xi\eta} = -\rho\beta\dot{v}$. This allows us to consider the wave interaction at the interface from the viewpoint of the theory of impact of rigid bodies [4].

Using the relations obtained above, we studied numerically the rearrangement of the surface of a shock-wave front at the interface between elastic media and the change in stresses for the reflected and refracted shock waves. It is shown that as the shock passes from a medium with smaller acoustic stiffness to a medium with greater acoustic stiffness, critical states arise at points H on the interface G at which the condition $\alpha_2\sin\theta_{1-} = \alpha_1$ is satisfied. In these states, the measure of geometrical divergence $RS \to 0$ at the front of the transmitted wave, and, therefore, $\dot{u}_{2+} \to \infty$ and $\varepsilon_{\xi\xi 2+} \to \infty$. Here R and S are the radii of curvature of the front surface in the corresponding directions [3] and $\varepsilon_{\xi\xi 2+}$ is the longitudinal strain of an element of the second medium. This property is related to the fact that the coefficient matrix of the left side of system (1.2) is degenerate and, even for a bounded right side, the system has a solution that goes to infinity.

Because of the unbounded growth of stresses near the points of QTIR, local stratification and failure of materials can occur in these regions in real stratified nonhomogeneous elastic media. However, an experimental verification of this effect is difficult because of the complexity of measuring stresses in a stratified body. Therefore, we performed experiments for the case where strain measurements are possible. The initial shock wave was generated in water, and Plexiglas was used as the elastic medium on which the shock wave was incident and which facilitated occurrence of the QTIR effect. Since water cannot resist shear forces, S-waves cannot appear in it. Therefore, we should set $\dot{v}_{1+} = 0$ in (1.2). Thus, only three equations for the three unknowns $\dot{u}_{1+}$, $\dot{u}_{2+}$, and $\dot{v}_{2+}$ are retained in the system.

The theoretical and experimental results were compared for the following physicomechanical parameters of the media: for water, $\rho_1 = 10^3$ kg/m^3 and $\alpha_1 = 1500$ m/sec; for Plexiglas, $\rho_2 = 1.12 \cdot 10^3$ kg/m^3, $E = 5.25 \cdot 10^9$ Pa, $\nu = 0.35$, and $\alpha_2 = 2500$ m/sec. We assumed that the spherical shock wave was generated by a point source in water at a distance of 0.3 m from the surface G. At the moment when it reached the point $x = 0$, $y = 0$ on the surface of the elastic plate, the stress at its front was $\sigma_{y1-} = 10^6$ Pa.

The procedure described above was used to calculate the stress-tensor components $\sigma_x = \sigma_{x2+}(x)$ (Fig. 2) and $\sigma_y = \sigma_{y2+}(x)$ (Fig. 3) at the point H in the plate as this point moved along the x axis on the plane G. One can see that the compressing stress $\sigma_{x2+}(x)$ increases monotonically and tends to infinity at the

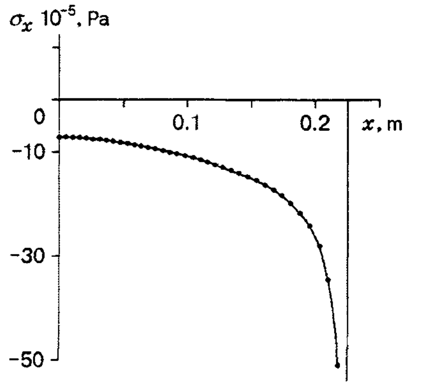

Fig. 2

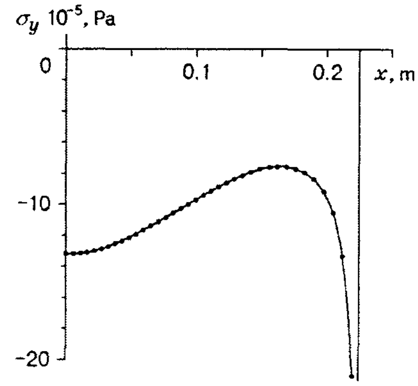

Fig. 3

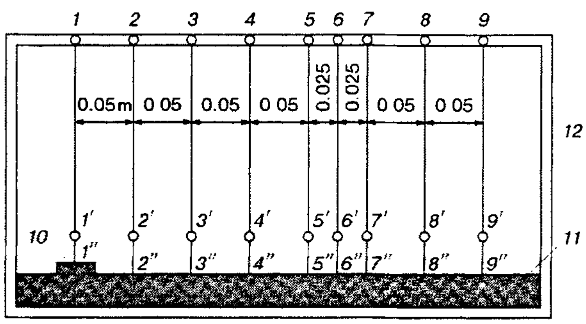

Fig. 4

point of QTIR $x = 0.225$ m, which corresponds to the angle of QTIR $\theta_{1-} = \arcsin(\alpha_1/\alpha_2) = 36.87°$. The function $\sigma_{y2+}(x)$ first decreases (Fig. 3). However, near the point of QTIR, it increases rapidly to infinity. For $x > 0.225$ m, the stress functions $\sigma_{x2+}(x)$ and $\sigma_{y2+}(x)$ were not calculated since the stress waves in the second medium are no longer shock waves.

2. Experimental Investigation of the QTIR Effect. For experimental investigations of mechanical wave phenomena in stratified media, we have improved the method described in [5, 6]. Since full-scale experiments are very expensive, primary attention was given to laboratory modeling of wave processes in layered media using electrical explosions. Since water transfers energy well and underwater electrical explosions are fairly safe, all laboratory experiments were performed in a cylindrical tank filled with a liquid. We used a cylindrical tank 3.2 m in diameter and 3 m high filled with water. For such a volume of water, the wave processes that take $2 \cdot 10^{-3}$ sec can be recorded in the center of the tank ignoring the effects of the walls and the free surface. Shock waves were generated by an electrical breakdown of a copper wire between the electrodes of a high-voltage cable connected to a high-voltage device made of four IKM-25-12 capacitors. The conductor explodes when a high voltage (up to 10 kV) is applied to the electrodes. For a working voltage of $2.5 \cdot 10^4$ V and a total capacity of $48 \cdot 10^{-6}$ F, the maximum energy accumulated in the capacitors can reach $1.5 \cdot 10^4$ J, which is equivalent to the energy of explosion of 2 g TNT.

This experimental facility produces shock waves that have a steep front (about 10^{-7} sec) and an exponential drop in postshock pressure. The time constant of the pressure drop for this facility is in the range $3 \cdot 10^{-5}$–$6 \cdot 10^{-5}$ sec, i.e., the pulses generated by the electrical explosions are sufficiently short and their propagation in stratified media can be described by the ray method [3].

Electrical explosion was produced in water at a distance of 0.3 m from a Plexiglas plate with dimensions 0.6×0.6 m and thickness 0.08 m, and it generated a pressure of 10^6 Pa in the epicenter of the incident shock wave. The physicomechanical parameters of the media were the same as those used above in theoretical calculations.

TABLE 1

x, m	$P_{\rm m} \cdot 10^{-5}$, Pa	$\varepsilon_{x2+}^{\rm m} \cdot 10^4$
0	10.6	0.20
0.05	10.2	−0.03
0.10	9.8	−0.66
0.15	10.5	−1.62
0.20	10.4	−3.80
0.225	9.9	−6.20
0.25	10.4	−8.80
0.30	10.4	−5.73
0.35	10.6	−3.20

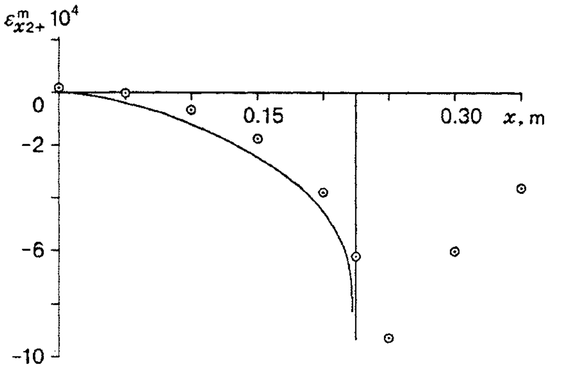

Fig. 5

The setting of the experiments on shock-wave propagation along the plane interface G between elastic media were complicated by the fact that the examined strain fields in both the liquid and solid media subjected to elastic deformation are discontinuous at four axisymmetric fronts intersecting on a circle that lies on the plane G and enlarges concentrically with time. To overcome the technical difficulties involved in the experimental registration of these fronts and measurement of the rapidly changing strain functions on them, we constructed a special case which could be used to move the source of electrical explosions in the water relative to the strain gauge attached to the Plexiglas plate. The experimental setup is shown in Fig. 4, where numbers 1–9 denote the points of successive initiation of explosions, $1'$–$9'$ denotes the location of the pressure transducers, 10 is the strain gauge, 11 is the Plexiglas plate, and 12 is the frame. An advantage of this setup is that successive displacement of the source of electrical explosions relative to the same strain gauge fixed on the plane G is equivalent to placing the strain gauge at various points $1''$–$9''$ on the surface G and a fixed position of the explosion source at the point 1. Since, in this case, we use the same strain gauge with the same method of attachment to the plate, it is possible to measure strains at various distances from the explosion source with the same error. The pressure transducers $1'$–$9'$ were also used to control the regularity of the position of the explosion source.

Strains were measured by a KTD-7B semiconductor strain gauge with a sensitivity coefficient of 100, a resistivity of 620 Ω, and a 0.007-m base, which was attached to the plate using ÉD-20 epoxy adhesive.

The pressure at the shock-wave front was measured by a pressure transducer made of TsTS-19 piezoelectric ceramics, which was connected to an AVK-3 coaxial antivibrator cable by the epoxy adhesive, which provided for reliable waterproofing and rigid fixation. This procedure allowed us to eliminate the cable effect and to measure the pressure at the shock-wave front with small error. The pressure measurements were recorded by one channel of a C9-16 double-beam oscillograph. The other channel was used to measure the strains ε_{x2+}.

Dynamic calibration of the transducers was performed using a shock tube. The experiments showed that the sensitivity of the pressure transducer made of TsTS-19 piezoelectric ceramics is linear in the pressure range 0–10^7 Pa. The total error of calibration of the pressure transducers was not greater than 10%.

Results of the experiments were entered into the computer and processed using the appropriate mathematical software.

We produced 33 electrical explosions by the procedure described above. Table 1 shows the distance x from the strain gauge to the explosion epicenter, the mean values of the readings of the control pressure transducer $P_{\rm m}$, and the mean values of the maximum relative compressing strains $\varepsilon_{x2+}^{\rm m}$ at the measurement points on the plate surface. The maximum strains are at a distance of 0.25 m from the explosion epicenter, and this differs from the theoretical results only by 10%. This difference can be explained by the experimental error and inaccuracy in measurements of the velocities α_1 and α_2 of longitudinal waves in elastic media. We also note that there is a difference between the theoretical strain $\varepsilon_{x2+}(0) = 0$ at the point $x = 0$ and the experimental tensile strain $\varepsilon^{\mathrm{m}}_{x2+}(0) = 0.2 \cdot 10^{-4}$ (see Table 1). This difference can be explained by direct impact of the pressure wave in water on its semiconducting element. The experimental value for the angle of QTIR is $\theta_{1-} = \arctan\,(0.25/0.3) = 39.79°$

For comparison, the theoretical strains (solid curve) and the strains averaged over a series of experiments (points) $\varepsilon^{\mathrm{m}}_{x2+}(x)$ are shown in Fig. 5. One can see that at a certain distance from the point of QTIR, these results are fairly close. The available difference can be explained by the fact that in the calculations we used a shock-wave profile in the form of an ideal step function and ignored the energy dissipation due to viscous friction in the materials. Allowance for these factors will probably lead to a better agreement between theoretical and experimental data.

In conclusion, we point out that the detected effect of unbounded growth of strains and stresses at points of QTIR at interfaces between elastic media is valid only within the framework of the theory of perfectly elastic bodies. In real continuous media, which are not perfectly elastic, this effect is manifested as a sharp increase in these parameters near the critical point.

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